Updated On 11-02-2025 By Lithanya

Chapter 10: Circles - NCERT Solutions for Class 10 Maths

Ex 10.1 Question 1.

How many tangents can a circle have?

Answer.

A circle can have infinitely many tangents since there are infinitely many points on the circumference of the circle and at each point of it, it has a unique tangent.

Ex 10.1 Question 2.

Fill in the blanks:
(i) A tangent to a circle intersects it in point(s).
(ii) A line intersecting a circle in two points is called a
(iii) A circle can have parallel tangents at the most.
(iv) The common point of a tangent to a circle and the circle is called

Answer.

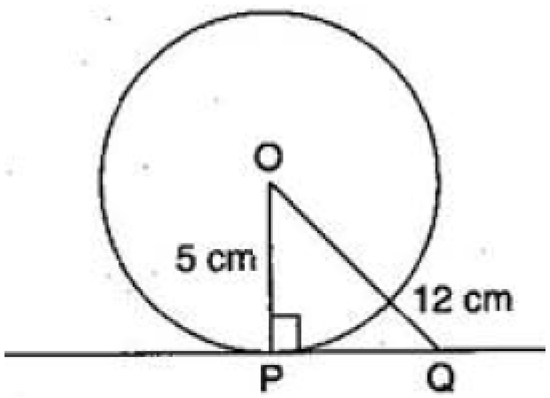
(i) A tangent to a circle intersects it in exactly one point.
(ii) A line intersecting a circle in two points is called a secant.
(iii) A circle can have two parallel tangents at the most.
(iv) The common point of a tangent to a circle and the circle is called point of contact.

Ex 10.1 Question 3.

A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that $OQ = 12 \text{ cm}$. Length PQ is:
(A) 12 cm
(B) 13 cm
(C) 8.5 cm
(D) $\sqrt{119} \text{ cm}$

Answer.

(D) $\because PQ$ is the tangent and OP is the radius through the point of contact.
$\therefore \angle \mathrm{OPQ} = 90^{\circ}$ [The tangent at any point of a circle is $\perp$ to the radius through the point of contact]
$\therefore$ In right triangle OPQ,



$OQ^2 = OP^2 + PQ^2$ [By Pythagoras theorem]

$\Rightarrow (12)^2 = (5)^2 + PQ^2$

$\Rightarrow 144 = 25 + PQ^2$

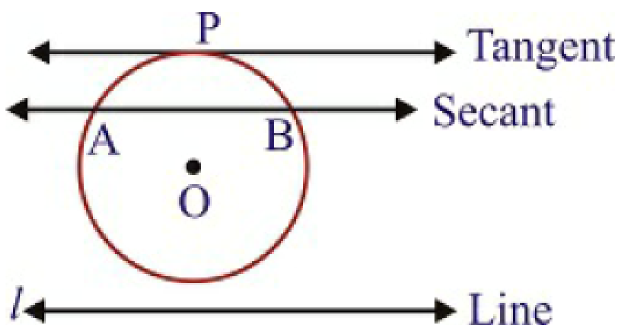
$\Rightarrow PQ^2 = 144 - 25 = 119$

$\Rightarrow PQ = \sqrt{119}$ cm

Ex 10.1 Question 4.

Draw a circle and two lines parallel to a given line such that one is a tangent and the other, a secant to the circle.

Answer:



Exercise 10.2 (Revised) - Chapter 10 - Circles - Ncert Solutions class 10 - Maths

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Chapter 10: Circles - NCERT Solutions for Class 10 Maths

Ex 10.2 Question 1.

From a point Q, the length of the tangent to a circle is $24 \mathrm{~cm}$ and the distance of Q from the centre is $25 \mathrm{~cm}$. The radius of the circle is:

(A) $7 \mathrm{~cm}$

(B) $12 \mathrm{~cm}$

(C) $15 \mathrm{~cm}$

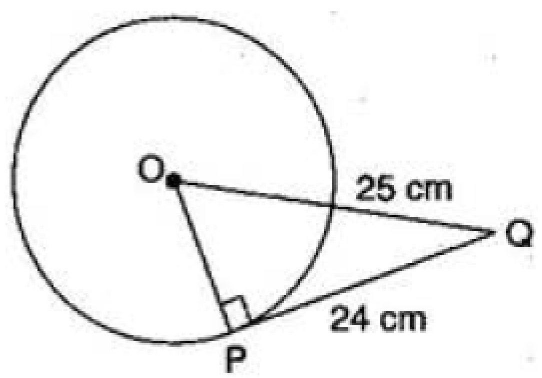
(D) $24.5 \mathrm{~cm}$

Answer.

(A)

$\because \angle \mathrm{OPQ}=90^{\circ}$

[The tangent at any point of a circle is to the radius through the point of contact]



$\therefore$ In right triangle OPQ,

$OQ^2=OP^2+PQ^2$

[By Pythagoras theorem]

$\Rightarrow(25)^2=OP^2+(24)^2$

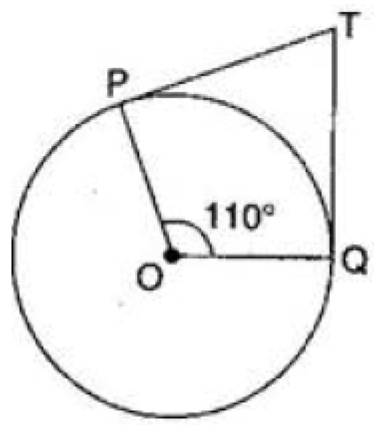
$\Rightarrow 625=OP^2+576$

$\Rightarrow OP^2=625-576=49$

$\Rightarrow OP=7 \mathrm{~cm}$

Ex 10.2 Question 2.

In figure, if TP and TQ are the two tangents to a circle with centre 0 so that $\angle POQ=110^{\circ}$, then $\angle$ PTQ is equal to:



(A) 60°
(B) 70°
(C) 80°
(D) 90°

Answer.

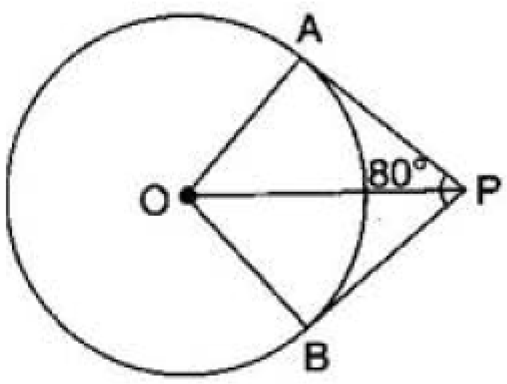
(B)
$\angle \mathrm{POQ}=110^{\circ}, \angle \mathrm{OPT}=90^{\circ}$ and $\angle \mathrm{OQT}=90^{\circ}$
[The tangent at any point of a circle is $\perp$ to the radius through the point of contact]
In quadrilateral OPTQ
$\angle \mathrm{POQ}+\angle \mathrm{OPT}+\angle \mathrm{OQT}+\angle \mathrm{PTQ}=360^{\circ}$
[Angle sum property of quadrilateral]
$\Rightarrow 110^{\circ}+90^{\circ}+90^{\circ}+\angle \mathrm{PTQ}=360^{\circ}$
$\Rightarrow 290^{\circ}+\angle \mathrm{PTQ}=360^{\circ}$
$\Rightarrow \angle \mathrm{PTQ}=360^{\circ}-290^{\circ}$
$\Rightarrow \angle \mathrm{PTQ}=70^{\circ}$

Ex 10.2 Question 3.

If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80°, then $\angle \mathrm{POA}$ is equal to:
(A) 50°
(B) 60°
(C) 70°
(D) 80°

Answer.

(A)
$\because \angle \mathrm{OAP}=90^{\circ}$
[The tangent at any point of a circle is $\perp$ to the radius
through the point of contact]



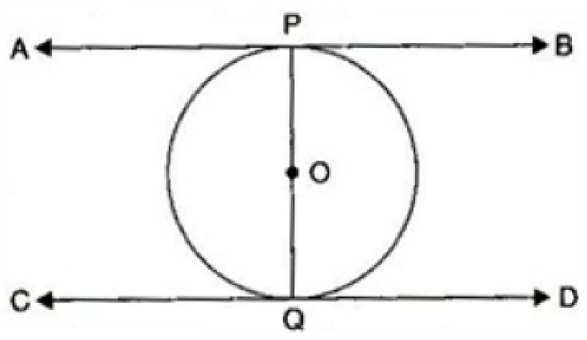
$\angle \mathrm{OPA}=\frac{1}{2} \angle \mathrm{BPA}=\frac{1}{2} \times 80^{\circ}=40^{\circ}$
[Centre lies on the bisector of the
angle between the two tangents]
In $\triangle \mathrm{OPA}$,
$\angle \mathrm{OAP}+\angle \mathrm{OPA}+\angle \mathrm{POA}=180^{\circ}$
[Angle sum property of a triangle]
$\Rightarrow 90^{\circ}+40^{\circ}+\angle \mathrm{POA}=180^{\circ}$
$\Rightarrow 130^{\circ}+\angle \mathrm{POA}=180^{\circ}$
$\Rightarrow \angle \mathrm{POA}=50^{\circ}$

Ex 10.2 Question 4.

Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Answer.

Given: PQ is a diameter of a circle with centre 0 .
The lines AB and CD are the tangents at P and Q respectively.



To Prove: $AB \| CD$

Proof: Since AB is a tangent to the circle at P and OP is the radius through the point of contact.

$\therefore \angle \mathrm{OPA}=90^{\circ}$

[The tangent at any point of a circle is $\perp$ to the radius through the point of contact]

$\because$ CD is a tangent to the circle at Q and OQ is the radius through the point of contact.

$\therefore \angle \mathrm{OQD}=90^{\circ}$

[The tangent at any point of a circle is $\perp$ to the radius through the point of contact]

From eq. (i) and (ii), $\angle \mathrm{OPA}=\angle \mathrm{OQD}$

But these form a pair of equal alternate angles also,

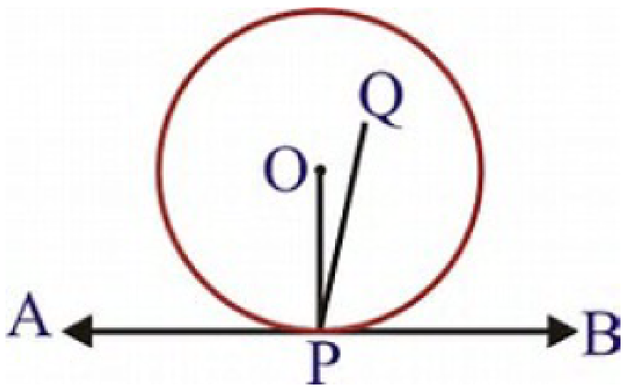
$\therefore AB \| CD$

Ex 10.2 Question 5.

Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

Answer.

Let AB be the tangent drawn at the point P on the circle with O.



If possible, let PQ be perpendicular to AB, not passing through O. Join OP.

Since tangnet at a point to a circle is perpendicular to the radius through the point.

Therefore, $\mathrm{AB} \perp \mathrm{OP} \quad \Rightarrow \quad \angle \mathrm{OPB}=90^{\circ}$

Also, $\angle \mathrm{QPB}=90^{\circ}$

[By construction]

Therefore, $\angle \mathrm{QPB}=\angle \mathrm{OPB}$, which is not possible as a part cannot be equal to whole.

Thus, it contradicts our supposition.

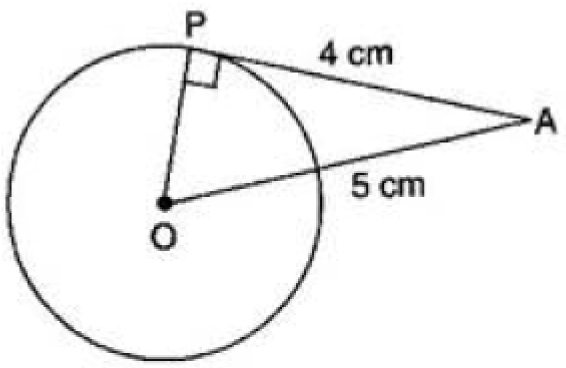
Hence, the perpendicular at the point of contact to the tangent to a circle passes through the centre.

Ex 10.2 Question 6.

The length of a tangent from a point A at distance $5 \mathrm{~cm}$ from the centre of the circle is $4 \mathrm{~cm}$. Find the radius of the circle.

Answer.

We know that the tangent at any point of a circle is $\perp$ to the radius through the point of contact.



$\therefore \angle \mathrm{OPA}=90^{\circ}$

$\therefore OA^{2}=OP^{2}+AP^{2}$

[By Pythagoras theorem]

$\Rightarrow(5)^{2}=(\mathrm{OP})^{2}+(4)^{2}$

$\Rightarrow 25=(\mathrm{OP})^{2}+16$

$\Rightarrow OP^{2}=9$

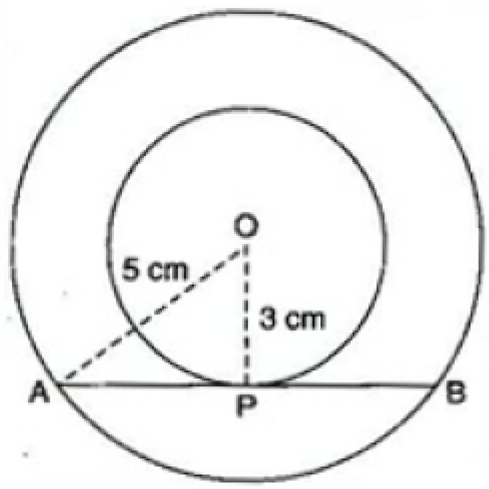
$\Rightarrow OP=3 \mathrm{~cm}$

Ex 10.2 Question 7.

Two concentric circles are of radii $5 \mathrm{~cm}$ and $3 \mathrm{~cm}$. Find the length of the chord of the larger circle which touches the smaller circle.

Answer.

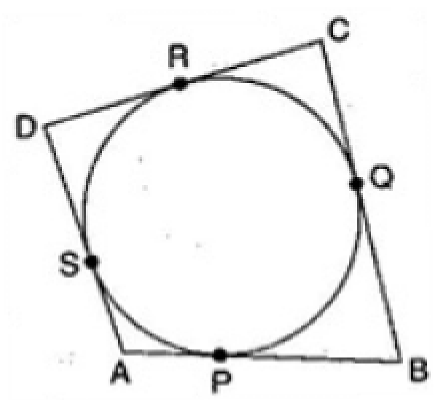
Let O be the common centre of the two concentric circles.



Let AB be a chord of the larger circle which touches the smaller circle at P.
Join OP and OA.
Then, $\angle \mathrm{OPA}=90^{\circ}$
[The tangent at any point of a circle is $\perp$ to the radius through the point of contact]
$\therefore \mathrm{OA}^{2}=\mathrm{OP}^{2}+\mathrm{AP}^{2}$
[By Pythagoras theorem]
$\Rightarrow(5)^{2}=(3)^{2}+A P^{2}$
$\Rightarrow 25=9+A P^{2}$
$\Rightarrow A P^{2}=16$
$\Rightarrow A P=4 \mathrm{~cm}$

Since the perpendicular from the centre of a circle to a chord bisects the chord, therefore
$\mathrm{AP}=\mathrm{BP}=4 \mathrm{~cm}$
$\Rightarrow \mathrm{AB}=\mathrm{AP}+\mathrm{BP}$
$=\mathrm{AP}+\mathrm{AP}=2 \mathrm{AP}$
$=2 \times 4=8 \mathrm{~cm}$
Ex 10.2 Question 8.

A quadrilateral ABCD is drawn to circumscribe a circle (see figure). Prove that:
$A B+C D=A D+B C$

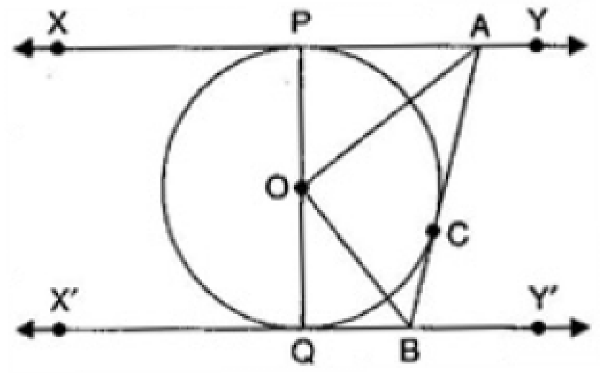


Answer.

We know that the tangents from an external point to a circle are equal.
$\therefore A P=A S$
$\mathrm{BP}=\mathrm{BQ}$
$C R=C Q$
$\mathrm{DR} = \mathrm{DS}$

On adding eq. (i), (ii), (iii) and (iv), we get
$(\mathrm{AP}+\mathrm{BP})+(\mathrm{CR}+\mathrm{DR})$
$=(\mathrm{AS}+\mathrm{BQ})+(\mathrm{CQ}+\mathrm{DS})$
$\Rightarrow \mathrm{AB}+\mathrm{CD}=(\mathrm{AS}+\mathrm{DS})+(\mathrm{BQ}+\mathrm{CQ})$
$\Rightarrow \mathrm{AB}+\mathrm{CD}=\mathrm{AD}+\mathrm{BC}$
Ex 10.2 Question 9.

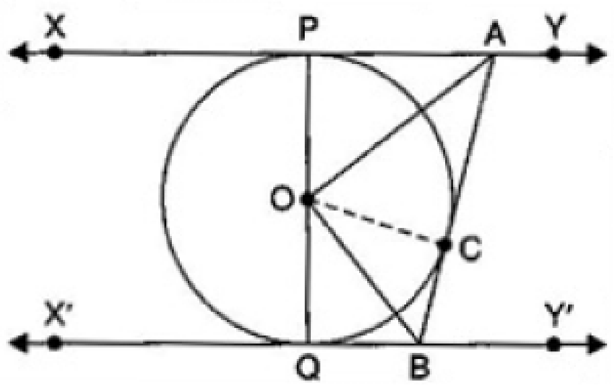
In figure, $X Y$ and $X^{\prime} Y^{\prime}$ are two parallel tangents to a circle with centre O and another tangent $A B$ with point of contact C intersecting $X Y$ at A and $X^{\prime} Y^{\prime}$ at B. Prove that $\angle A O B=90^{\circ}$.



Answer.

Given: In figure, XY and $X'Y'$ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and $X'Y'$ at B.

To Prove: $\angle AOB = 90^\circ$
Construction: Join OC
Proof: $\angle OPA = 90^\circ$ .... (i)



$\angle OCA = 90^\circ$
[Tangent at any point of a circle is $\perp$ to the radius through the point of contact]

In right angled triangles OPA and OCA,
$\angle OPA = \angle OCA = 90^\circ$
$OA = OA$ [Common]
$AP = AC$ [Tangents from an external

point to a circle are equal]
$\therefore \triangle OPA \cong \triangle OCA$
[RHS congruence criterion]
$\therefore \angle OAP = \angle OAC$[By C.P.C.T.]
$\Rightarrow \angle OAC = \frac{1}{2}\angle PAB$
Similarly, $\angle OBQ = \angle OBC$
$\Rightarrow \angle OBC = \frac{1}{2}\angle QBA$
$\because$ XY \| X'Y' and a transversal AB intersects them.
$\therefore \angle PAB + \angle QBA = 180^\circ$
[Sum of the consecutive interior angles on the same side of the transversal is 180°]
$\Rightarrow \frac{1}{2}\angle PAB + \frac{1}{2}\angle QBA$
$= \frac{1}{2} \times 180^\circ$
$\Rightarrow \angle OAC + \angle OBC = 90^\circ$
[From eq. (iii) \& (iv)]
In $\triangle AOB$,
$\angle OAC + \angle OBC + \angle AOB = 180^\circ$
[Angel sum property of a triangle]

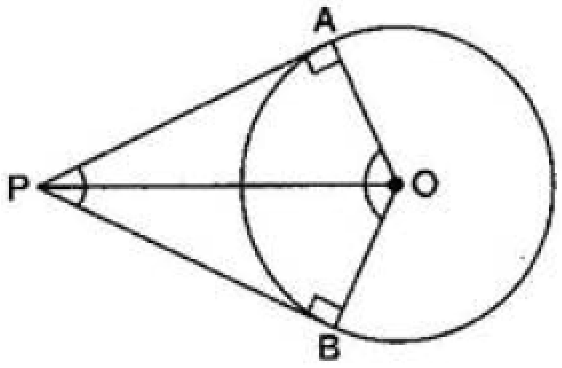
$\Rightarrow 90^\circ + \angle AOB = 180^\circ$[From eq. (v)]
$\Rightarrow \angle AOB = 90^\circ$

Hence proved.
Ex 10.2 Question 10.

Prove that the angel between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

Answer.

$\angle OAP = 90^\circ$
$\angle OBP = 90^\circ$.   (ii)



[Tangent at any point of a circle is $\perp$ to
the radius through the point of contact]
$\because$ OAPB is quadrilateral.
$\therefore \angle \mathrm{APB}+\angle \mathrm{AOB}+\angle \mathrm{OAP}+\angle \mathrm{OBP}=360^{\circ}$
[Angle sum property of a quadrilateral]
$\Rightarrow \angle \mathrm{APB}+\angle \mathrm{AOB}+90^{\circ}+90^{\circ}=360^{\circ}$
[From eq. (i) \& (ii)]
$\Rightarrow \angle \mathrm{APB}+\angle \mathrm{AOB}=180^{\circ}$
$\therefore \angle \mathrm{APB}$ and $\angle \mathrm{AOB}$ are supplementary.
Ex 10.2 Question 11.

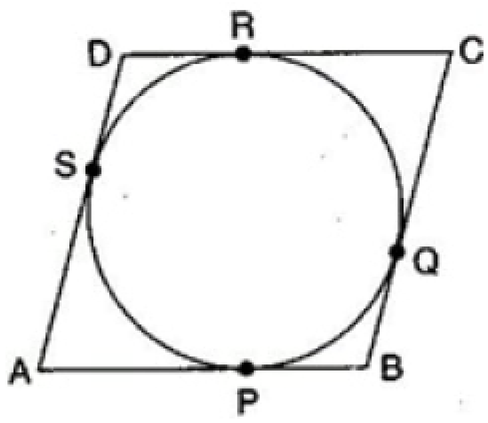
Prove that the parallelogram circumscribing a circle is a rhombus.

Answer.

Given: ABCD is a parallelogram circumscribing a circle.

To Prove: ABCD is a rhombus.

Proof: Since, the tangents from an external point to a circle are equal.
$\therefore \mathrm{AP}=\mathrm{AS}$



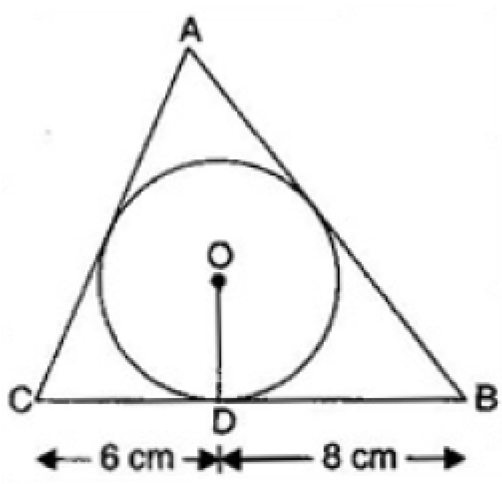
$BP=BQ$
$CR=CQ$
$DR=DS$

On adding eq. (i), (ii), (iii) and (iv), we get
$(\mathrm{AP}+\mathrm{BP})+(\mathrm{CR}+\mathrm{DR})$
$=(\mathrm{AS}+\mathrm{BQ})+(\mathrm{CQ}+\mathrm{DS})$
$\Rightarrow \mathrm{AB}+\mathrm{CD}=(\mathrm{AS}+\mathrm{DS})+(\mathrm{BQ}+\mathrm{CQ})$
$\Rightarrow \mathrm{AB}+\mathrm{CD}=\mathrm{AD}+\mathrm{BC}$
$\Rightarrow \mathrm{AB}+\mathrm{AB}=\mathrm{AD}+\mathrm{AD}$
[Opposite sides of \| gm are equal]
$\Rightarrow 2 \mathrm{AB}=2 \mathrm{AD}$
$\Rightarrow \mathrm{AB}=\mathrm{AD}$

But $\mathrm{AB}=\mathrm{CD}$ and $\mathrm{AD}=\mathrm{BC}$

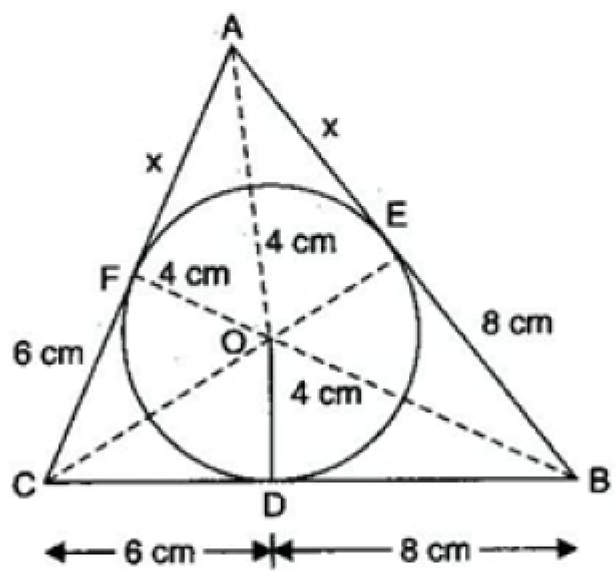
[Opposite sides of \| gm]
$\therefore \mathrm{AB}=\mathrm{BC}=\mathrm{CD}=\mathrm{AD}$
$\therefore$ Parallelogram ABCD is a rhombus.
Ex 10.2 Question 12.

A triangle ABC is drawn to circumscribe a circle of radius $4 \mathrm{~cm}$ such that the segments BD and DC into which BC is divided by the point of contact D are of lengths $8 \mathrm{~cm}$ and $6 \mathrm{~cm}$ respectively (see figure). Find the sides AB and AC.



Answer.

Join OE and OF. Also join OA, OB and OC.



Since $\mathrm{BD} = 8 \mathrm{~cm}$
$\therefore \mathrm{BE} = 8 \mathrm{~cm}$

[Tangents from an external point to a circle are equal]
Since $CD = 6 \mathrm{~cm}$
$\therefore \mathrm{CF} = 6 \mathrm{~cm}$
[Tangents from an external point to a circle are equal]
Let $\mathrm{AE} = \mathrm{AF} = x$
Since $OD = OE = OF = 4 \mathrm{~cm}$
[Radii of a circle are equal]

$$\therefore \text{ Semi-perimeter of } \Delta \mathrm{ABC} = \frac{(x+6)+(x+8)+(6+8)}{2} = (x+14) \mathrm{cm}$$

$$\therefore \text{ Area of } \Delta \mathrm{ABC} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{(x+14)(x+14-14)(x+14-\overline{x+8})(x+14-\overline{x+6})}$$

$$= \sqrt{(x+14)(x)(6)(8)} \mathrm{cm}^2$$

Now, Area of $\triangle \mathrm{ABC} =$ Area of $\triangle \mathrm{OBC} +$ Area of $\triangle OCA +$ Area of $\triangle OAB$

$$\Rightarrow \sqrt{(x+14)(x)(6)(8)}$$

$$= \frac{(6+8)4}{2} + \frac{(x+6)4}{2} + \frac{(x+8)4}{2}$$

$$\Rightarrow \sqrt{(x+14)(x)(6)(8)}$$

$$= 28 + 2x + 12 + 2x + 16$$

$$\Rightarrow \sqrt{(x+14)(x)(6)(8)} = 4x + 56$$

$$\Rightarrow \sqrt{(x+14)(x)(6)(8)} = 4(x+14)$$

Squaring both sides,
$(x+14)(x)(6)(8) = 16(x+14)^2$
$\Rightarrow 3x = x + 14$
$\Rightarrow 2x = 14$
$\Rightarrow x = 7$
$\therefore \mathrm{AB} = x + 8 = 7 + 8 = 15 \mathrm{~cm}$

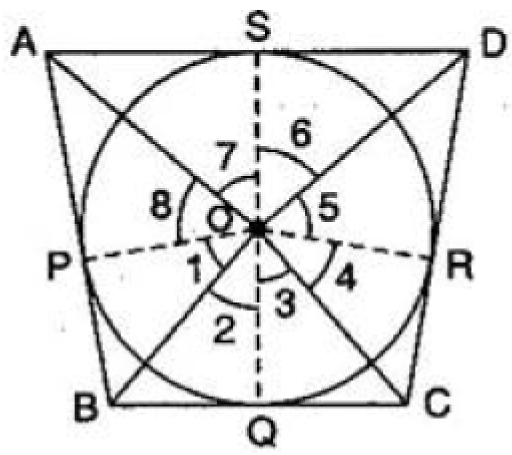
And $\mathrm{AC} = x + 6 = 7 + 6 = 13 \mathrm{~cm}$

Ex 10.2 Question 13.

Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Answer.

Given: ABCD is a quadrilateral circumscribing a circle whose centre is O.
To prove: (i) $\angle \mathrm{AOB}+\angle \mathrm{COD}=180^{\circ}$ (ii) $\angle \mathrm{BOC}+\angle \mathrm{AOD}=180^{\circ}$
Construction: Join OP, OQ, OR and OS.



Proof: Since tangents from an external point to a circle are equal.

$\therefore \mathrm{AP}=\mathrm{AS}$,

$\mathrm{BP}=\mathrm{BQ} \ldots$

$CQ=CR$

$DR=DS$

In $\triangle \mathrm{OBP}$ and $\triangle \mathrm{OBQ}$,
$\mathrm{OP}=\mathrm{OQ}$ [Radii of the same circle]
$\mathrm{OB}=\mathrm{OB}$ [Common]
$\mathrm{BP}=\mathrm{BQ}[$ [From eq. (i)]
$\therefore \Delta \mathrm{OPB} \cong \Delta \mathrm{OBQ}$ [By SSS congruence criterion]
$\therefore \angle 1=\angle 2[$ By c.P.C.T.]
Similarly, $\angle 3=\angle 4, \angle 5=\angle 6, \angle 7=\angle 8$
Since, the sum of all the angles round a point is equal to 360°.

$\therefore \angle 1+\angle 2+\angle 3+\angle 4+\angle 5+\angle 6+\angle 7+\angle 8=360^{\circ}$

$\Rightarrow \angle 1+\angle 1+\angle 4+\angle 4+\angle 5+\angle 5+\angle 8+\angle 8=360^{\circ}$

$\Rightarrow 2(\angle 1+\angle 4+\angle 5+\angle 8)=360^{\circ}$

$\Rightarrow \angle 1+\angle 4+\angle 5+\angle 8=180^{\circ}$

$\Rightarrow(\angle 1+\angle 5)+(\angle 4+\angle 8)=180^{\circ}$

$\Rightarrow \angle \mathrm{AOB}+\angle \mathrm{COD}=180^{\circ}$

Similarly, we can prove that
$\angle \mathrm{BOC}+\angle \mathrm{AOD}=180^{\circ}$